Report on the Hausdorff Trimester Program

Diophantine Equations

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The study of Diophantine equations is one of the oldest branches of pure mathematics, and is still flourishing. In a way the basic problems can be formulated in terms of school mathematics like the Fermat problem: What are the integer solutions of the equation $x^n + y^n = z^n$ for a fixed integer n. The 20th century saw great progress: the proof of Siegel's theorem on integer points of algebraic curves, the negative solution of Hilbert's 10th problem, the proof of Mordell's conjecture by Faltings, and the proof of Fermat's Last Theorem by Wiles.

The program intended to bring together specialists from different areas of the theory of Diophantine equations and to provide an excellent opportunity for interactions between them. One of the slightly unusual aspects of the program was that experts from number theory were mixed with people who have their background more in logic. This was particularly fruitful.

The atmosphere during the program was very lively. This had a lot to do with the right mixture of people and the involvement of the Max Planck Institute, which was not only represented by one of the organizers, but also by the active participation of several of its visitors.

Besides the organizers an large group of prominent mathematicians could be attracted, amongst them Jörg Brüdern, Jean-Louis Colliot-Thélène, Ehud Hrushovski, Björn Poonen, Florian Pop, Alexandra Shlapentokh and Yuri Tschinkel. These people interacted very well with the impressive group of young mathematicians. This interaction was supported by two seminars, the Oberseminar Diophantische Gleichungen and the Seminar in number theory.

To mention a few topics which were covered by the program, here are some examples of talks given during the program:
- Zeros of three p-adic quadratic forms
- A generalization of the Bombieri-Pila determinant method
- On rational points of homogeneous spaces

- Mordell-Weil problem for cubic surfaces: in search of a model-theoretic approach
- Motivic Poisson Summation
- On solubility of Diophantine equations in p-adic numbers

Numerous papers emerged from the program. To give an example, Jean-Louis Colliot-Thélène and Olivier Wittenberg proved that the equation $x^3 + y^3 + 2z^3 = a$ has no Brauer-Manin obstruction over the integers.

A very successful workshop was organized at the end of the program. The program was - besides by many papers - documented in a proceedings which in a way can serve as an introduction to the modern theory of Diophantine equations.